Name: _____

Activity – Related Rates

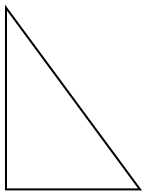
Related Rates problems involve rates of change of variables that are changing over time. Remember that rates of change are derivatives. Implicit differentiation must be used as time is not a specified variable within the equation.

Guidelines for Solving a Related Rate Problem:

1. Draw a diagram, and clearly label the variable quantities and known quantities.
2. Write down the *given* values of the rates of change of the variables with respect to time *t*.
3. Find an equation that relates the variables.
4. Differentiate both sides of this equation implicitly with respect to *t*.
5. Replace the variables and derivative in the resulting equation by the values found in step 2, and solve this equation for the required unknown rate of change.

Problem: A 10-ft ladder rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/sec, how fast is the ladder sliding down the wall at any time *t*? How fast is the ladder sliding down the wall at the specific instant when the bottom of the ladder is 6 ft from the wall?

i) Begin by labeling the diagram showing the situation. Let *x* be the length along the wall and let *y* be the distance from the bottom of the ladder to the wall. Label the hypotenuse, which represents the ladder with its given length.



ii) Use the Pythagorean Theorem to write an equation that shows the relationship between the variables in the diagram.

iii) Now consider which values are changing, and what their rates of change would be. In this case we were given the rate of change of *y* with respect to time, and are asked to find the rate of change of *x* with respect to time. Using correct notation, write down the rate of change of *y* with respect to time.

iv) Using implicit differentiation, find the derivative with respect to time of both sides of the equation in part ii.

v) To find the rate at which the ladder is sliding down the wall at time *t*, substitute in the constant rate of change of *y* and solve for $\frac{dx}{dt}$.

vi) How fast is the ladder sliding down the wall at the specific instant when the bottom of the ladder is 6 ft from the wall? Note: it is important to remember that the ladder is moving. Do not substitute the constant lengths that occur at this one instant in time at the beginning of the problem or all the rates of change will be zero!

Complete the following problems using the guidelines provided.

1. Car A is traveling west at 50 miles/hour and car B is traveling north at 60 miles/hour. Both are headed for the intersection of the two roads. At what rate is the distance between the cars changing when car A is 0.3 miles and car B is 0.2 miles from the intersection?

2. A man walks along a straight path at a speed of 4 feet/second. A searchlight is located on the ground 20 feet from the path and is kept focused on the man. At what rate is the search light rotating when the man is 15 feet from the point on the path closest to the searchlight?

3. A spotlight on the ground shines on a wall 12 meters away. If a man 2 meters tall walks from the spotlight towards the building at a speed of 1.6 meters/second, how fast is the length of his shadow on the building decreasing when he is 4 meters from the building?

4. Two ships leave the same port at noon. Ship A is headed due north at 18 km/hr. Ship B is headed N45°E at 20 km/hr. How fast is the distance between the ships changing at 12:30 pm? (Hint: a trigonometric law will be needed.)

5. Challenge Problem: A water tank has the shape of an inverted circular cone (point down) with a base of radius of 6 feet and a depth of 8 feet. Water is being pumped into the tank at a constant rate of 4 cubic feet per minute. At what rate is the depth of the water changing when the water in the tank is 3 feet deep?

   Complete the first three steps in the guidelines above.

   Make a list of each of the variables in your equation. The rate of change of which variable is given? The rate of change of which variable is going to be the solution to the problem? Do you see a potential problem here?

   Can a variable be eliminated from your equation? If so, which variable would you choose and why? Are you given enough information to eliminate this variable? If possible, eliminate the variable and proceed to solve the problem.