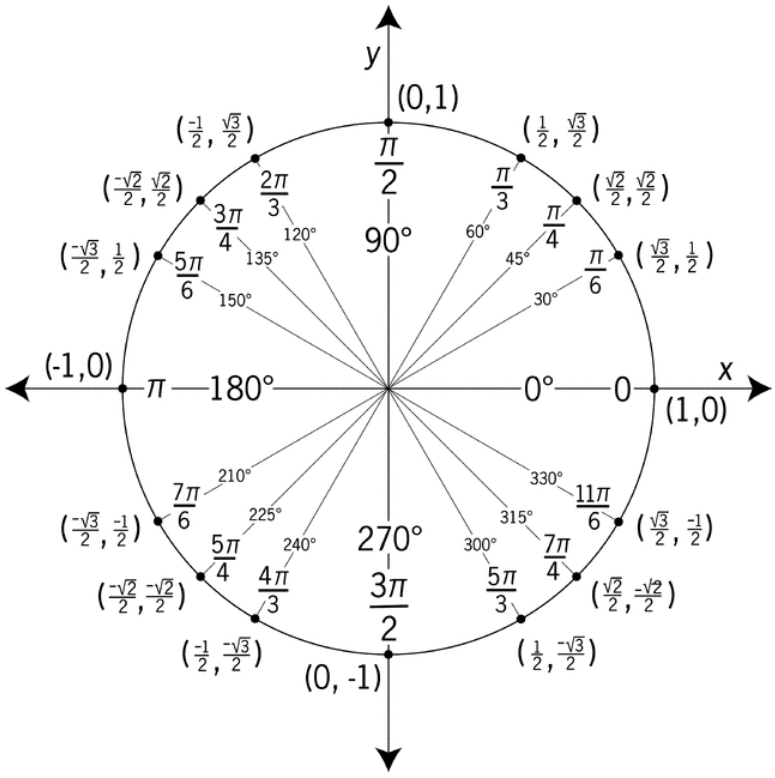
Name_____

Activity – Solving Trigonometric Equations

Unless directed otherwise, give all solutions in terms of radians.

Part 1. Using the Unit Circle to Solve Basic Cosine and Sine Equations

1. Consider the equation $\sin x = \frac{1}{2}$. This is asking for what values of *x*, the sine equals $\frac{1}{2}$.



   a) Think of the unit circle. On the unit circle there are two points where the sine is $\frac{1}{2}$. What are the angles for these two points?

   b) Are these the only angles that have a sine value of $\frac{1}{2}$?

   c) List all *x* values where the sine is $\frac{1}{2}$.

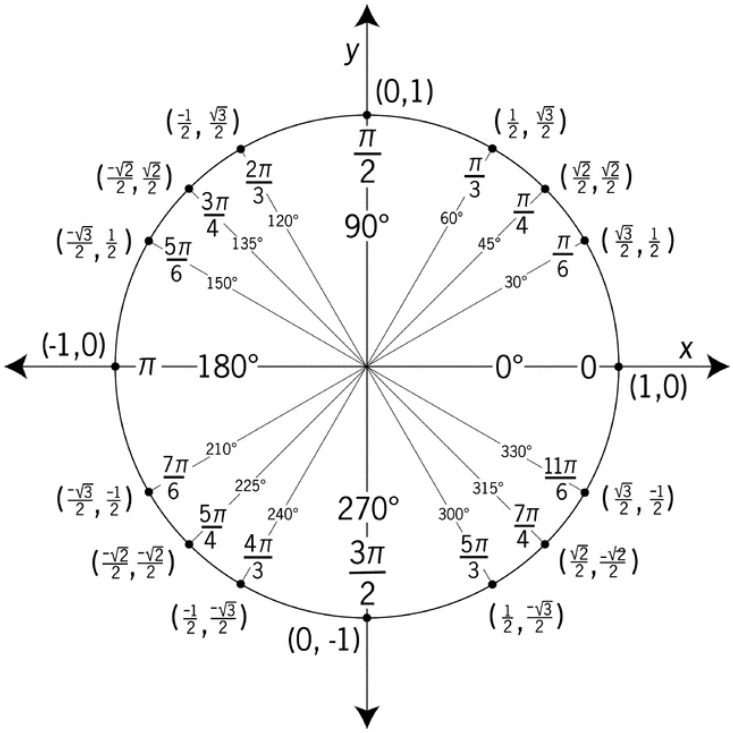
2. Consider the equation $\cos x = -\frac{1}{2}$.

   a) On the unit circle there are two points where the cosine is $-\frac{1}{2}$. What are the angles for these two points?

   b) List all *x* values where the cosine is $-\frac{1}{2}$.

3. Consider the equation $\cos x = -\frac{\sqrt{2}}{2}$. List all solutions to this equation.

Part 2. Using the Unit Circle to Solve Reciprocal and Quotient Trig Function Equations



1. Consider the following equations. First write the reciprocal equation and then use the unit circle to find all solutions where $0 \le x < 2\pi$.

a) $\sec x = -\dfrac{2}{\sqrt{3}}$

b) $\csc x = -1$

c) $\sec x = -2$

d) $\sec x = -\sqrt{2}$

e) $\csc x = 2$

2. Remember that $\tan x = \dfrac{\sin x}{\cos x}$ and $\cot x = \dfrac{\cos x}{\sin x}$.

Solve the following equations where *x* is any real number.

a) $\tan x = -1$

b) $\cot x = -1$

c) $\tan x = -\sqrt{3}$

d) $\cot x = \dfrac{1}{\sqrt{3}}$

Part 3. Using Algebra and Using a Calculator to Solve Trig Equations

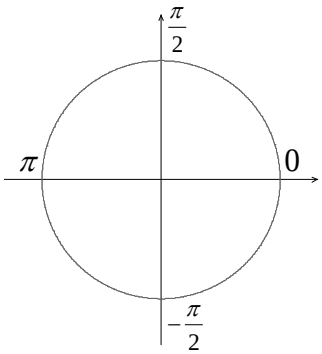
1. If you have a trig equation that just one trig function, you should first do some algebra to isolate the trig function and then think of what angles make the equation true. Do this on the following giving all solutions where $0 \le x < 2\pi$.

a) $2\sin x = -1$

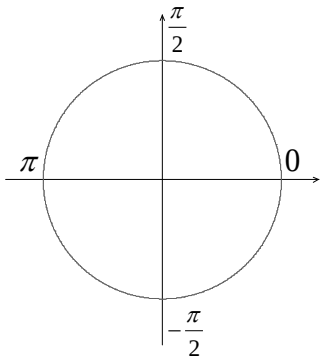
b) $2\cos x = \sqrt{3}$

2. We can use a calculator to solve trig equations where the solutions are not known values on the unit circle. HOWEVER, remember for inverse trig functions that the range is restricted so your calculator will only give you values in that range though there could be other solutions as well.

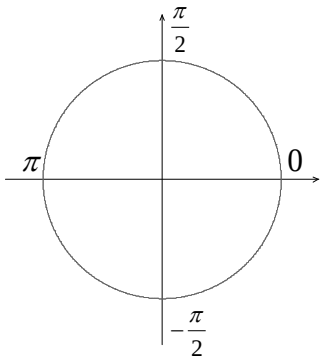
a) To help you remember those, shade the range on each unit circle below.



$y = \cos^{-1} x$



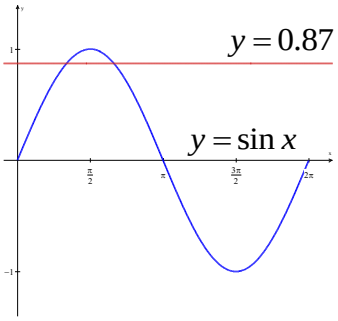
$y = \sin^{-1} x$



$y = \tan^{-1} x$

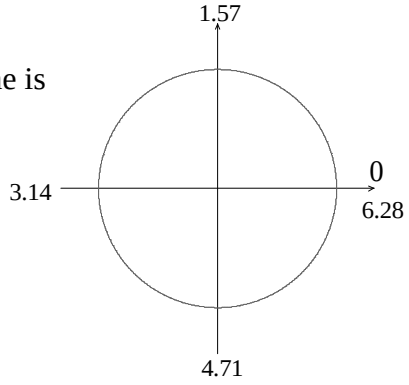
b) To solve the equation $\sin x = 0.87$ on $0 \le x < 2\pi$, you can use the $\sin^{-1}$ button. This will often involve hitting a yellow or blue button first and then the sin button. What is the angle whose sine is 0.87?

BUT, this will only give you one solution to the equation and there are 2 places that have the same value on $0 \le x < 2\pi$. Let's look at both a Cartesian graph and a unit circle to see this.

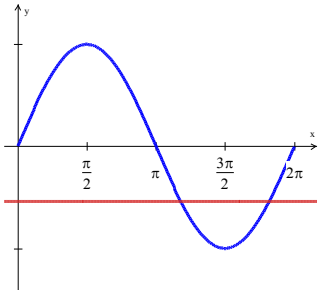


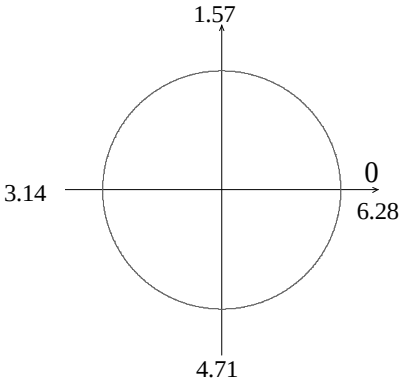
On the unit circle at right, draw an angle whose sine is approximately 0.87. Draw another angle on the circle that has the same sine value.

Determine the measure of this angle.



c) For $\sin x = -0.54$ on $0 \le x < 2\pi$ there will again be two answers as the graph shows. Mark those 2 angles on the unit circle and determine their values.





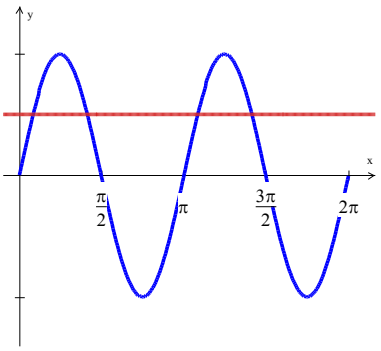
Part 4. Looking at Changes in ω

1. What if the variable in our trig function has a coefficient? How will this affect our answers? Let's see.

   a) Consider $\cos(2x)=\frac{1}{2}$ on $0 \le x < 2\pi$. . What does that 2 do to the graph?

   So how many solutions would you expect?

Look at the graph to confirm your answer above.



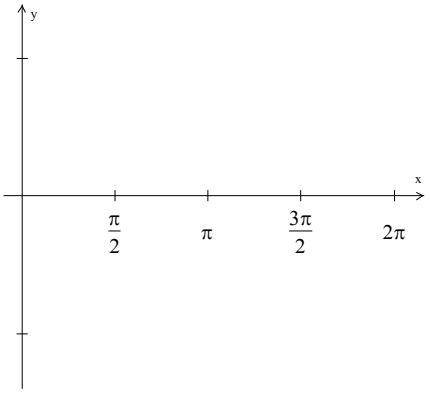
Let's solve the equation $\cos(\theta)=\frac{1}{2}$.

In your solution, replace θ with $2x$ and solve for x.

This will give you two solutions, but you need four. How can you figure out the other two?

   b) Now consider $\cos(3x)=\frac{1}{2}$ on $0 \le x < 2\pi$. Draw a graph here similar to the one above that illustrates our new equation and solutions. How many solutions will there be?

   Find those solutions.



   c) How many solutions do you think $\cos(20x)=\frac{1}{2}$ on $0 \le x < 2\pi$ will have?

Part 5. Solving Trig Equations by Factoring

1. Solve the following equations by factoring.

   a) $2u^2 - u = 0$

   b) $2u^2 + 3u + 1 = 0$

   c) $u^2 + 3u - 4 = 0$

2. Now solve the following trig equations by factoring. Your factoring in #1 will help.

a) $2\sin^2 x - \sin x = 0$

b) $2\cos^2 x + 3\cos x + 1 = 0$

c) $\sin^2 x + 3\sin x - 4 = 0$

3. We may first need to apply a trig identity before we can factor or solve (you can't forget those identities---because now we need them ☺). Often it is useful to get the trig equation in terms of only one type of trig function or so that they have the same argument (angle). Try using an identity first on the following equations.

a) $\sin^2 x = 2\cos x$

b) $\sin 2x + \cos x = 0$

c) $\cos 2x - \cos x = 0$