MATH 1010 LAB ACTIVITY – Linear Programming Project Prep

The purpose of this lab is to introduce you to the technique of linear programming and help you get started with your second group project.

OPTIMIZATION

Many real-world problems have to do with obtaining the best result within a given set of constraints or restrictions. This best result might be a greatest value (a maximum) or a least value (a minimum).

For example, a business might like to know how to make the *most* profit with the *least* expense possible. Or they might want to *minimize* their loss. For a factory, the managers are interested in *maximizing* productivity while also *minimizing* costs. However each of these situations has constraints that must be accounted for like the budget, the number of employees, the production capacity of the plant, space available, etcetera.

Some of these types of problems (called optimization problems) can be solved using systems of inequalities and a technique called linear programming.

Some concepts you may need to review:

1. Graphing systems of linear inequalities, section 4.4
2. Solving systems of linear equations, section 3.2

EXAMPLE – SOLVE USING LINEAR PROGRAMMING

Say you own a lumber company and you can convert logs either into lumber or plywood. In any given week, you can turn out 400 units of production total. You have regular customers that require 100 units of lumber and 150 units of plywood each week.

You are trying to make the most profit you can each week. Your profit on a unit of lumber is $20 and on a unit of plywood is $30. How many units of lumber and plywood should you make each week in order to maximize your weekly profit?

We'll follow a 7-step process to solve linear programming problems:

1. Define variables.
2. Write an objective function (the function to be maximized or minimized.)
3. Write a system of inequalities using the constraints given in the problem.
4. Graph the system of inequalities.
5. Find the coordinates of the vertices of the feasible region (the shaded area in your graph of the inequalities).
6. Substitute the coordinates of the vertices into the objective function and evaluate.
7. Select the greatest or least result to answer the problem

For the lumber problem, we need to find how many units of lumber and how many units of plywood. For step 1 Define variables, we do the following:

x = the number of units of lumber to produce each week

y = the number of units of plywood to produce each week

We want to maximize our weekly profit and we make $20 for selling a unit of lumber and $30 for selling a unit of plywood, so the function we want to maximize is

Profit = money from lumber + money from plywood

Profit = $20x + $30 y

This function is our **objective function** for step 2.

Now we need to write inequalities using the constraints in the problem. We are only able to create a total of 400 units of production each week, so we know that the number of units of lumber plus the number of units of plywood must sum to no more than 400 units. We can write this as:

$$x + y \leq 400$$

We know that we have to produce at least 100 units of lumber for our regular customers. That inequality would be

$$x \geq 100$$

Also we must produce 150 units of plywood each week for our regular customers:

$$y \geq 150$$

That gives us a total of 3 inequalities that represent all of these restrictions or constraints. When we write all the **constraints** (the inequalities) together with the **objective function** (the profit function we wrote) this is called a **linear programming problem**.
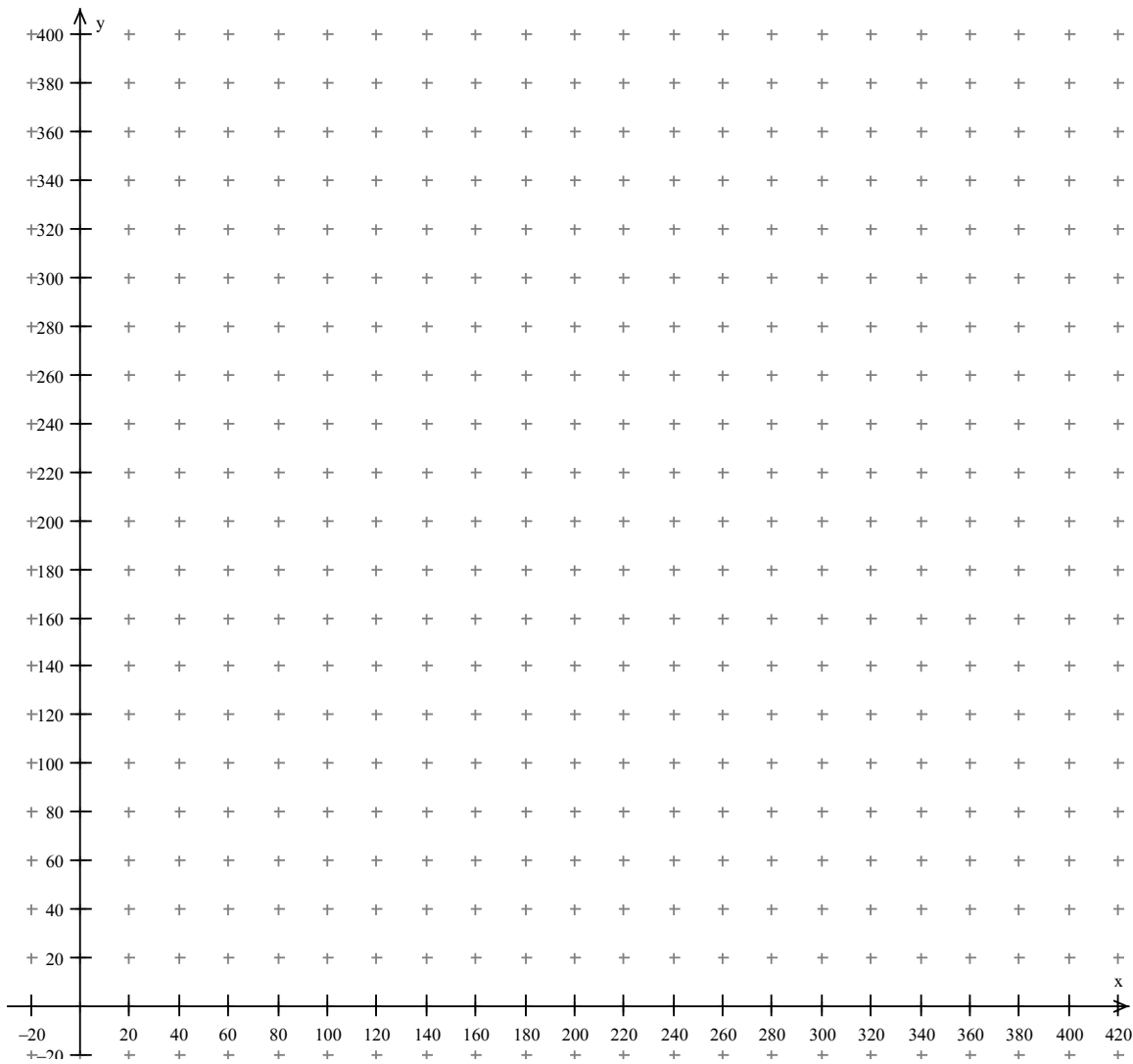
$$x + y \leq 400$$

$$x \geq 100$$

$$y \geq 150$$

Profit = $20x + $30y

To solve a system of inequalities we graph the solution set like we did in section 4.4.

1. Work with a partner to graph the three inequalities on the following graph (next page), and shade the intersection of all three. Remember to label the x and y axes with words to indicate what the variables represent.

You can see that your shaded area is shaped like a triangle. This shaded area is called the **feasible region**. Points in that area are potential solutions to this problem of how much lumber and how much plywood to make.

For example, notice that the point (180, 180) is in the feasible region. This would represent producing 180 units of lumber and 180 units of plywood each week. That meets all the constraints (go look at them to verify!) and also meets our restriction on total number of units produced per week.

It turns out that the **corners** of this region are the potential combinations that may maximize our objective function (our maximum profit per week). So first we need to find the coordinates of the corners. For this problem they are **(100, 300)**, **(250, 150)**, and **(100, 150).** Be sure you can verify this algebraically!

To figure-out which is the best one, we plug each solution into our **objective function** (the profit function), then select the solution that gives us the largest profit.

$$\text{Profit} = \$20x + \$30y$$

2. Evaluate the profit for the first corner: (100,300). This is when we manufacture 100 units of lumber and 300 units of plywood in a week. What is the profit for that week?

3. Now evaluate the profit function at the corner (250, 150).

4. Finally evaluate the profit function at the corner (100, 150).

5. How many units of lumber and plywood should you make each week in order to maximize your weekly profit? (Remember to write a complete sentence.)

6. Now reflect on how you used the 7-step process to solve this optimization problem. Will you be able to apply it to a new problem?

To see more examples of linear programming problems and their solutions, see chapter 4.5 of your textbook.

GROUP PROJECT 2: LINEAR PROGRAMMING PROJECT

Group Size: 3 to 4 people. One person is not a group and any projects submitted by individual students will not be graded.

Due Date: _____

Get in a group. Each group member needs to fill out the information below. Once you have each others' names and contact information, ask your teacher for a copy of the project.

Group names and contact information.

1.

2.

3.

4.

Now you're ready to work on the project. As you are working, you can refer to section 4.5 for examples and help, and here are a couple of YouTube videos with solutions.

http://www.youtube.com/watch?v=vAjYgl3kXjQ

http://www.youtube.com/watch?v=2ACJ9ewUC6U